THE DYNAMICS OF HEATING THE ELEMENTS OF A SELF-STARTED POWER UNIT

V. S. Ermakov and G. I. Khutskii

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A method is presented for the construction of transient responses in the vapor heating of metallic components of a power unit on starting. An analytical relationship is derived to relate overregulation in a system to the period between the separate instants at which the rates of temperature rise in the metal are measured.

To avoid intolerable thermal stresses in the vapor heating of metal components of complex configuration in a power unit (a boiler, a turbine) protected by insulation, the rate of the temperature rise in the metal must be regulated at a number of points. Since the time for the heating of the components, as a rule, must be kept to a minimum, automatic devices are called upon to maintain the maximum permissible rate of temperature rise. This rate serves as the decisive parameter in the automatic control of the metal-heating process.

The temperature rise at selected points is monitored periodically and elements of lag therefore necessarily appear in the automatic-control system. However, the existence of one or more elements exhibiting lag in an automatic system, as is well known, results in a transcendental equation for the control system. Estimates of the overregulation make it possible to clarify the extent to which basic characteristics of individual elements of the control system, including the delay elements, have been properly selected.

As is well known, metal heating may be described by differential equations compiled on the basis of a heat balance, and these are of the form

$$\gamma_m V_m c_m \frac{dT_m}{dt} = \alpha_{v,m} F_{v,m}(T_v - T_m) - \frac{\lambda_i}{\delta_i} F_{m,i}(T_m - T_i), \qquad (1)$$

$$\gamma_i V_i c_i \frac{dT_i}{dt} = \frac{\lambda_i}{\delta_i} F_{m,i}(T_m - T_i) - \alpha_{i,a} F_{i,a}(T_i - T_a). \qquad (2)$$

Equations (1) and (2) have been written in analogy with the differential equations of Campbell, cited in [1]. Campbell notes that in a number of cases we can adopt an elementary approach to the problem of dynamics, involving the assumption that the rate of the temperature rise in the body is proportional to the total quantity of heat transmitted to the body by convection, conduction, and radiation:

$$\rho c_p V \frac{dT}{dt} = H_{conv} + H_{cond} + H_{rad}.$$

This nonlinear equation in certain cases is easily linearized. For example, in the case of heat transfer by heat conduction the rate of change in the average temperature T for the volume of a material is defined from the differential equation

$$c \frac{dT}{dt} = G(T_{av} - T).$$

As demonstrated in our research, the Campbell equations are more convenient for the practical calculations of the systems being analyzed than is an equation of the form

$$c\gamma \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2}$$

having the corresponding boundary conditions.

Having solved Eqs. (1) and (2) according to Laplace, we find that

$$T_m(p) = \frac{k}{Tp+1} \Big\{ \alpha_{v,m} F_{v,m} T_v(p) + \frac{\lambda_i}{\delta_i} F_{m,i} [T_i(p) - T_m(p)] +$$

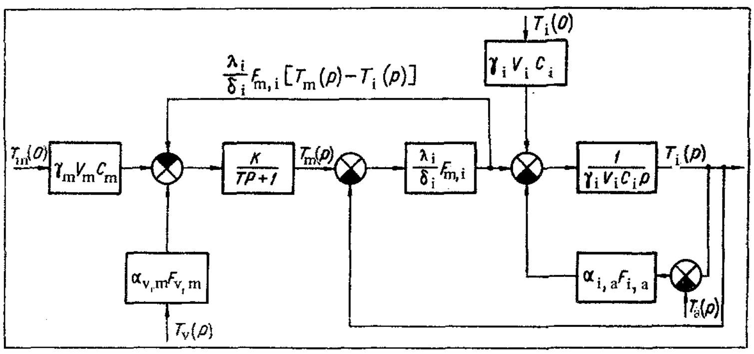

Fig. 1. Control system of controlled object.

$$+ \gamma_m V_m c_m T_m(0) \Big\} , \quad (3)$$

$$T_i(p)\gamma_i V_i c_i p = \frac{\lambda_{ii}}{\delta_i} F_{m\,i}[T_m(p) - T_i(p)] +$$

$$+ \alpha_{i,a} F_{i,a}[T_a(p) - T_i(p)] +$$

$$+ \gamma_i V_i c_i T_i(0). \quad (4)$$

Using expressions (3) and (4), we construct a control system for the controlled object (Fig. 1).

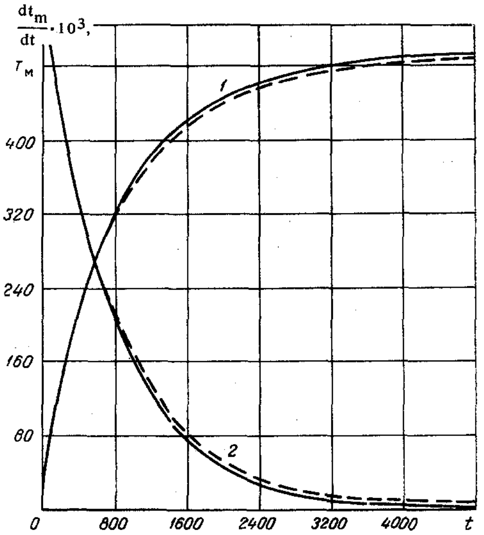

Fig. 2. Comparison of electromodeling curves; 1) time variation in temperature of metal on heating (°C)(solid line); 2) time variation of temperature growth rate (°C/sec) (dashed line, assumptions $T_i(0) = 0$ and $T_a(0) = 0$).

The modeling of this circuit on an MN-7 electronic analog computer demonstrated that the specified values of the initial insulation temperature $T_i(0)$ and the ambient temperature $T_a(0)$ have but a slight effect on the nature of the process involved in the heating of the metal. Indeed, comparison of the electronic simulation curves (Fig. 2) shows that the maximum error resulting from neglect of these terms does not exceed 2%. This made it possible to simplify the control circuit of the object, thus facilitating the analysis of the operation for the entire system of heating-rate control.

The assumptions $T_i(0) = 0$ and $T_a(0) = 0$ in the Laplace-transform differential equations of the object make it possible to derive a transfer function convenient for practical calculations.

The control system for the heating-rate control of the metal is shown in Fig. 3 where the controlled object has been arbitrarily divided into two parts. The input quantity for the I-st part of the object is the vapor flow rate G, while the output quantity is the vapor temperature T_v. The latter serves as the input quantity for the II-nd part of the object at whose output the rate of change in the metal temperature is measured: $V = dT_m/dt$.

The circuit includes a link in which there is delay and this, as indicated earlier, is brought about by the measurement of the quantity V at discrete intervals of time which are functions of the design of the scanning device.

The following equations can be set up for the individual links of the system:

1) the relay element

$$y = \begin{cases} k_r \text{ when } x > 0, \\ 0 \text{ when } x \leqslant 0; \end{cases} \quad (5)$$

2) the electric motor

$$T_i \frac{dg}{dt} = y; \quad (6)$$

3) the regulating valve

$$G = k_1 g; \quad (7)$$

4) the I-st part of the object

$$T_v = kG; \quad (8)$$

5) the II-nd part of the object

$$\frac{\gamma_m V_m c_m}{\alpha_{v\cdot m} F_{v\cdot m}} \frac{dT_m}{dt} + T_m = T_v$$

when $T_m = T_i$ with consideration of the fact that the quantities $T_i(0)$ and $T_a(p)$ have been neglected. Having differentiated this expression and having carried out a Laplace transformation, we have

$$(Tp + 1)V(p) = pT_v(p); \quad (9)$$

6) the measuring device

$$W_{m.d}(p) = k_i; \quad (10)$$

7) the delay link

$$W_\tau(p) = \exp[-\tau p]. \quad (11)$$

It follows from the structural diagram (Fig. 3) that the transfer function of the linear portion of the control system can be written as follows:

$$W(p) = W_1(p) W_{val}(p) W_{I\,p.o}(p) \times$$
$$\times W_{II\,p.o}(p) W_{m.d.}(p) W_\tau(p). \quad (12)$$

Having substituted the values of the transfer functions for the separate elements of the system to the right-hand part of expression (12), we obtain

$$W(p) = \frac{k}{Tp+1} \exp[-\tau p]. \quad (13)$$

Considering the specific features of the relay element [2], we find that the linear portion of the system is subject to rectangular pulses of constant amplitude whose sign, duration, and relative position are functions both of the external effect and of the state of the linear portion of the system. In the general case, the parameters of the pulses are functions of the control signal and of the threshold values of the relay element. The control effect is constant between the adjacent switching times t_k and t_{k+1}.

With actuation of the relay on attainment of the maximum rate of temperature rise, the lifting of the

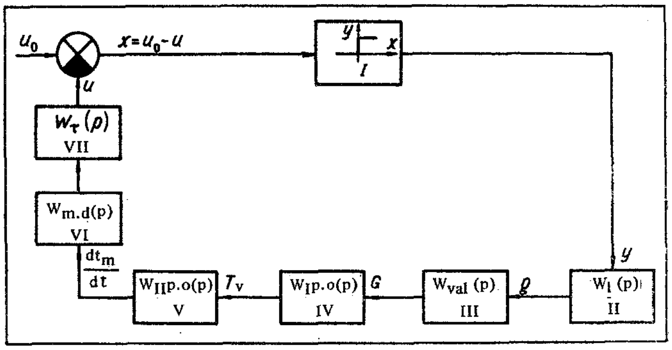

Fig. 3. Control system of heating rate control: I) relay element; II) electromotor; III) regulating valve; IV) first part of object; V) second part of object; VI) measuring device; VII) delay link.

regulating valve in the control system under consideration ceases. No provision is made for reverse operation of the valve to avoid a delay in the heating time. In this connection, the static characteristic of the relay element in the system under consideration is described by Eq. (5) rather than by the equation $y = k_r \operatorname{sign} x$.

For the given relay element we can write the transform of a control effect of duration $t_{k+1} - t_k$, beginning with the instant t_k, as

$$\frac{k_r}{p}\left(\frac{\exp[-pt_k] - \exp[pt_{k+1}]}{2}\right) +$$

$$+ (-1)^r \frac{k_p}{p}\left(\frac{\exp[-pt_k] - \exp[-pt_{k+1}]}{2}\right).$$

The inverse transform of the regulated quantity z(t) is given in the form of a sum of the components

$$z(t) = z_1(t) + z_2(t), \qquad (14)$$

where

$$z_1(t) = k_r\left[\frac{1}{2} h(t) + \sum_{k=1}^{n} (-1)^k h(t - t_k)\right]; \qquad (15)$$

$$z_2(t) = \frac{k_p}{2} h(t). \qquad (16)$$

From expressions (14), (15), and (16) we obtain the final value for the controlled quantity in expanded form:

$$z(t) = \begin{cases} k_r h(t) & \text{when } 0 < t < t_1, \\ k_r h(t) - k_r h(t - t_1) & \text{when } t_1 < t < t_2, \\ k_r h(t) - k_r h(t - t_1) + k_r h(t - t_2) & \text{when } t_2 < t < t_3. \end{cases} \qquad (17)$$

It follows from expression (17) that to calculate z(t), i.e., to construct the transient response, it is necessary, first of all, to calculate the time characteristic of the linear portion of the system and to determine the switching times t_k. The calculation of the time characteristic h(t) of the linear portion of the system is possible by analytical, graphical, and similar methods. The switching times t_k which are roots of the equation $x(t_k) = 0$ are most conveniently determined graphically, with the simultaneous construction of the transient response z(t).

For the system under consideration, the time characteristic is calculated by means of the expansion formula

$$h(t) = \frac{P(0)}{Q(0)} + \sum_{\nu=1}^{n} \frac{P(p_\nu)}{Q^1(p_\nu)\, p_\nu} \exp[p_\nu t],$$

where

$$W(p) = \frac{P(p)}{Q(p)}.$$

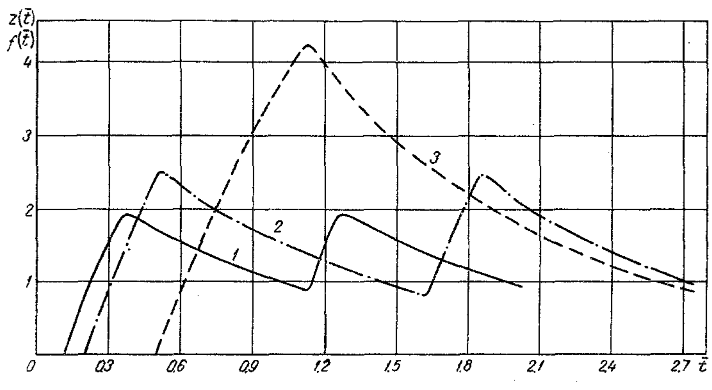

Fig. 4. Curves of transient processes in heating: 1) τ = 2 min; T = 17 min; 2) 2 and 10; 3) 2 and 4.

With introduction of the dimensionless magnitudes of the instantaneous time $\bar{t} = t/T$ and the time of pure delay $\tau = \tau/T$, the time characteristic $h(\bar{t})$ assumes the form

$$h(t) = K\{1 - \exp[-(\bar{t} - \bar{\tau})]\}. \quad (18)$$

In constructing the transient responses we employ the Tsypkin method [2].

Figure 4 shows the transient responses constructed for the case in which the external effect applied to the input of the relay element at the instant $\bar{t} = 0$ changes in jumpwise fashion and then remains constant and equal to unity, i. e., $f(\bar{t}) = 1$ when $\bar{t} \geq 0$.

An examination of the transient responses for various values of the time constant T is made necessary by the fact that the quantity T diminishes with increasing temperature as a result of the increase in the heat-transfer coefficient $\alpha_{v,m}$. The curves show that as T diminishes the overregulation of the quantity z(t) gradually increases, becoming more than 4 times as great when T = 4 min. Thus, from the construction of the transient responses we see the effect exerted on the dynamics of the system by the continuously varying time constant T.

It is easy to see that the overregulation will increase even if the time τ of pure delay increases when T = = const, i. e., with an increase in the period of complete sweep of the scanning device. Indeed, the greater the scanning cycle, the longer the system remains without the effect of negative feedback, i. e., the parameter being regulated is somewhat beyond control and attains a correspondingly greater magnitude during this period. In the general case, the overregulation in the system is a function of the relative magnitude $\bar{\tau} = \tau/T$.

The solution of the reverse problem is of interest in the design of an automatic control system, i. e., the evaluation of the maximum possible period between the instants at which the measurements are carried out on the basis of the specified overregulation magnitude. Therefore, we must obtain an analytical function relating the relative overregulation $\bar{\Pi}$ and the relative delay $\bar{\tau}$.

As follows from the evaluation of the transient responses, the first maximum of overregulation of the controlled parameter occurs at an instant of time defined as

$$\bar{t} = 2\bar{\tau} + \bar{t}_{(\bar{t}_1 - \bar{\tau})}, \quad (19)$$

where $\bar{t}_{(\bar{t}_1 - \bar{\tau})}$ is the time during which the controlled parameter, varying from zero, reaches a value equal in magnitude to the external effect (in our case, $f(\bar{t})$ = = 1).

On elapse of the delay time $\bar{\tau}$, the controlled parameter varies according to the law $z(t) = k_r h(t)$ all the way to the instant at which the first overregulation maximum sets in, and this always occurs in the interval $t_1 < t < t_2$.

With (18) and (19) we obtain

$$\bar{\Pi} = K\{1 - \exp[-\bar{t}_1]\}. \quad (20)$$

The time $\bar{t}_{(\bar{t}_1 - \bar{\tau})}$, as we can see from the curves of the transient responses, is constant in all cases, and therefore

$$K\{1 - \exp[-\bar{t}_{(\bar{t}_1 - \bar{\tau})}]\} = 1$$

and

$$\bar{t}_{(\bar{t}_1 - \bar{\tau})} = \ln\frac{K}{K-1}. \quad (21)$$

Having substituted the value of the time $\bar{t}_{(\bar{t}_1 - \bar{\tau})}$ into expression (20), by means of simple transformations we find

$$\bar{\Pi} = K\left\{1 - \exp\left[-\left(\ln\frac{K}{K-1} + \bar{\tau}\right)\right]\right\},$$

$$\bar{\tau} = \ln\frac{K-1}{K-\bar{\Pi}}. \quad (22)$$

The derived formula relates the relative delay in the system with the relative overregulation in the transient response. By means of this formula we determine the required cycle for the complete sweep of the scanning device, given a known magnitude for the time constant T in the case of a specified relative overregulation $\bar{\Pi}$ in the system, and we also evaluate the possible overregulation in the control system for the metal-heating process.

The chosen control system for which the problem of evaluating the possible overregulation has been solved may be regarded as a typical automatic control system since it exhibits a simple structural diagram and includes generally accepted units and links.

NOTATION

γ_m and γ_i are the specific weights of the metal and the insulation, respectively; c_m and c_i are the specific heat capacities of the metal and the insulation; T_v, T_m, T_i, and T_a are the temperatures of vapor, metal, insulation, and surrounding air, respectively; $F_{v,m}$, $F_{m,i}$, and $F_{i,a}$ are the surfaces with heat flux from vapor to metal, from metal to insulation, and from insulation to surrounding air, respectively; λ_i is the thermal conductivity coefficient of insulation; δ_i is the thickness of the insulation; $\alpha_{v,m}$ and $\alpha_{i,a}$ are the heat transfer coefficients from vapor to metal and from insulation to air.

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The Belorussian Branch of the Krzhizhanovskii Power Engineering Institute, Minsk